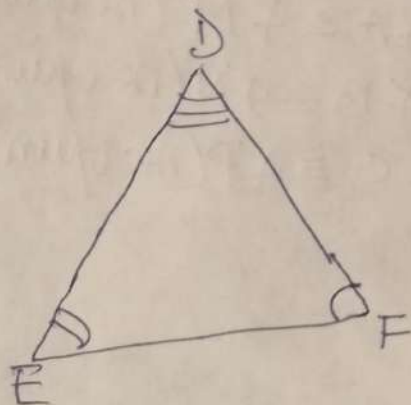
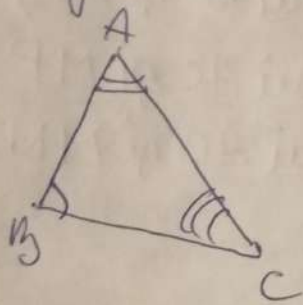


Criterii de asemănare a două triunghiuri

Știm că 2 triunghiuri sunt asemenea dacă au unghiurile congruente 2 câte 2 și laturile proporționale

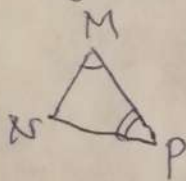
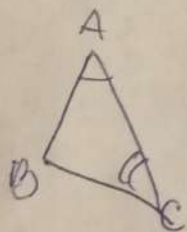


$$\begin{aligned} \sphericalangle A &\equiv \sphericalangle D \\ \sphericalangle B &\equiv \sphericalangle E \\ \sphericalangle C &\equiv \sphericalangle F \end{aligned}$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF}$$

În practică, pentru a demonstra asemănarea a 2 triunghiuri, avem 3 teoreme (criterii) care ne permit să demonstrăm mai ușor asemănarea celor 2 triunghiuri.

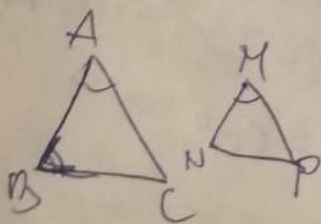
Criteriul U.U. - Două triunghiuri care au 2 perechi de unghiuri congruente sunt asemenea



$$\begin{aligned} \sphericalangle A &\equiv \sphericalangle M \\ \sphericalangle C &\equiv \sphericalangle P \end{aligned} \left. \vphantom{\begin{aligned} \sphericalangle A &\equiv \sphericalangle M \\ \sphericalangle C &\equiv \sphericalangle P \end{aligned}} \right\} \text{U.U.} \Rightarrow \triangle ABC \sim \triangle MNP$$

$$\Rightarrow \frac{AB}{MN} = \frac{AC}{MP} = \frac{BC}{NP}$$

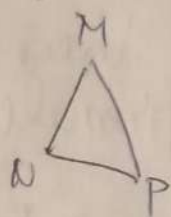
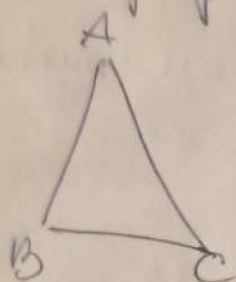
Criteriul L.U.L. - Două triunghiuri care au 2 perechi de laturi proporționale și unghiurile dintre ele congruente sunt asemenea.



$$\begin{aligned} \frac{AB}{MN} &= \frac{AC}{MP} \\ \sphericalangle A &\equiv \sphericalangle M \end{aligned} \left. \vphantom{\begin{aligned} \frac{AB}{MN} &= \frac{AC}{MP} \\ \sphericalangle A &\equiv \sphericalangle M \end{aligned}} \right\} \text{L.U.L.} \Rightarrow \triangle ABC \sim \triangle MNP$$

$$\Rightarrow \frac{AB}{MN} = \frac{AC}{MP} = \frac{BC}{NP}$$

Criteriul L.L.L. Două triunghiuri care au laturile proporționale sunt asemenea



$$\frac{AB}{MN} = \frac{AC}{MP} = \frac{BC}{NP} \stackrel{LLL}{\Rightarrow} \Delta ABC \sim \Delta MNP$$

$$\Rightarrow \begin{aligned} \sphericalangle A &\equiv \sphericalangle M \text{ (opun lui BC și NP)} \\ \sphericalangle B &\equiv \sphericalangle N \text{ (opun lui AC și MP)} \\ \sphericalangle C &\equiv \sphericalangle P \text{ (opun lui AB și MN)} \end{aligned}$$

Aplicații

1) ΔABC
 $\Delta A'B'C'$

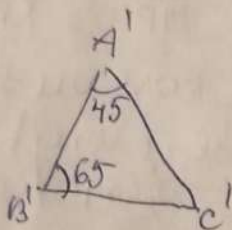
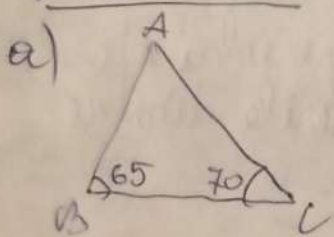
a) $m(\sphericalangle B) = 65^\circ$
 $m(\sphericalangle C) = 70^\circ$
 $m(\sphericalangle A') = 45^\circ$
 $m(\sphericalangle B') = 65^\circ$

$$\Delta ABC \sim \Delta A'B'C'$$

b) $\frac{AB}{A'B'} = \frac{BC}{B'C'}$
 $m(\sphericalangle B) = 60^\circ$
 $m(\sphericalangle A') = 42^\circ$
 $m(\sphericalangle C') = 78^\circ$

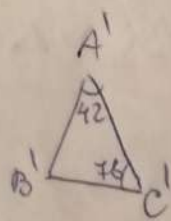
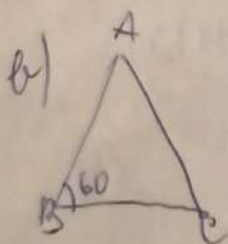
$$\Delta ABC \sim \Delta A'B'C'$$

Rezolvare



$$m(\sphericalangle A) = 180 - (65 + 70) = 45^\circ$$

Deci $\sphericalangle A \equiv \sphericalangle A'$
 $\sphericalangle B \equiv \sphericalangle B'$ $\left. \begin{array}{l} \\ \\ \end{array} \right\} \stackrel{UU}{\Rightarrow} \Delta ABC \sim \Delta A'B'C'$



$$m(\sphericalangle B') = 180 - (42 + 78) = 60^\circ$$

Deci $\sphericalangle B \equiv \sphericalangle B'$ $\left. \begin{array}{l} \\ \\ \end{array} \right\} \stackrel{LUL}{\Rightarrow} \Delta ABC \sim \Delta A'B'C'$

$$\frac{AB}{A'B'} = \frac{BC}{B'C'}$$

2) $\triangle ABC$
 $\triangle MNP$

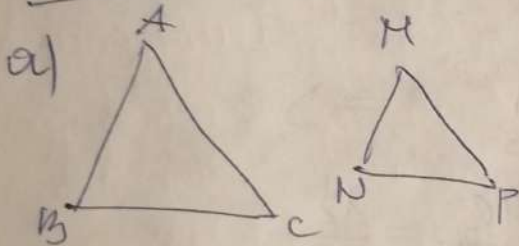
a) $AB = \frac{4}{5} MN$
 $AC = 0,8 MP$
 $BC = 80\% NP$

Verificati dacă
 $\triangle ABC \sim \triangle MNP$

b) $m(\sphericalangle A) = 0,6 \cdot 90^\circ$; $m(\sphericalangle B) = 70^\circ$; $m(\sphericalangle P) = 80^\circ$; $\left. \begin{array}{l} \triangle ABC \sim \\ \triangle MNP \end{array} \right\}$
 $m(\sphericalangle M) = 0,3 \cdot 180^\circ$

c) $\sphericalangle N = \sphericalangle B$
 $\frac{AB}{3} = \frac{MN}{4}$
 $\frac{BC}{18} = \frac{NP}{24}$ $\left. \begin{array}{l} \triangle ABC \sim \triangle MNP \end{array} \right\}$

Rezolvare

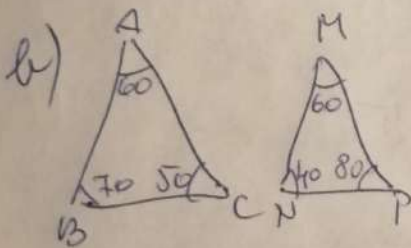


$$AB = \frac{4}{5} \cdot MN \quad | : MN \Rightarrow \frac{AB}{MN} = \frac{4}{5}$$

$$AC = 0,8 \cdot MP \quad | : MP \Rightarrow \frac{AC}{MP} = 0,8 = \frac{8}{10} = \frac{4}{5}$$

$$BC = 80\% NP \quad | : NP \Rightarrow \frac{BC}{NP} = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

$\frac{LLL}{\Rightarrow} \triangle ABC \sim \triangle MNP$



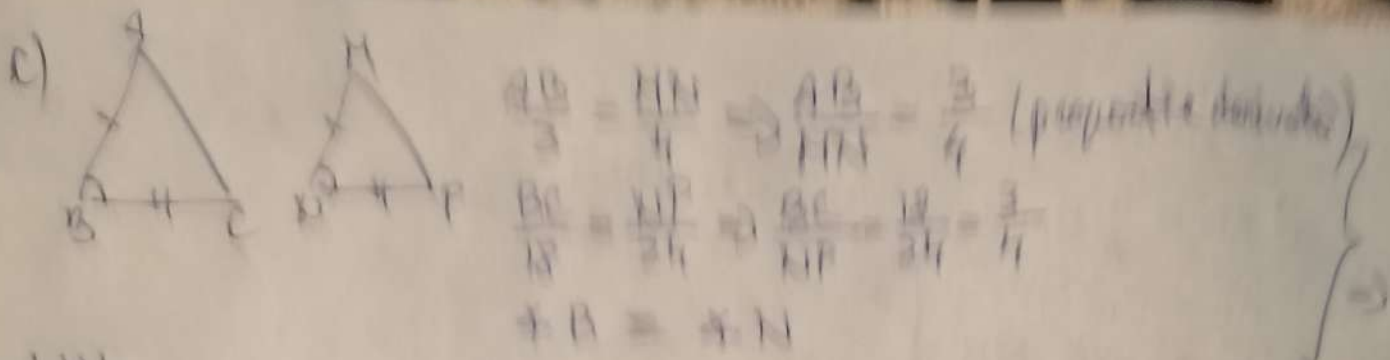
$$m(\sphericalangle A) = \frac{6}{9} \cdot 90 = 60^\circ$$

$$m(\sphericalangle M) = \frac{3}{9} \cdot 180 = 60^\circ$$

$$m(\sphericalangle N) = 180 - (60 + 80) = 40^\circ$$

$$m(\sphericalangle C) = 180 - (60 + 70) = 50^\circ$$

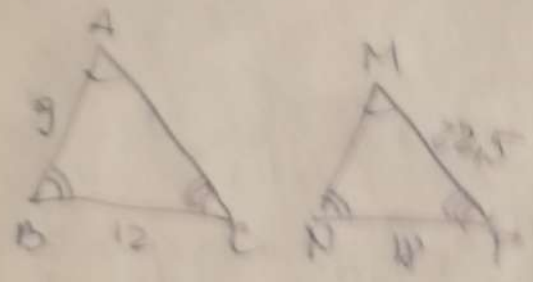
~~XXXXXXXXXX~~
 $\Rightarrow \triangle ABC$ nu e asemenea cu $\triangle MNP$.



LVL
 $\Rightarrow \Delta ABC \sim \Delta MNP$

3) $\Delta ABC \sim \Delta MNP$

$AB = 9$ $BC = 12$ $NP = 18$ $MP = 22,5$	$AC = 15$ $MN = 8$
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$\Delta ABC \sim \Delta MNP \Rightarrow$
 $\sphericalangle A = \sphericalangle M$
 $\sphericalangle B = \sphericalangle N$
 $\sphericalangle C = \sphericalangle P$

$$\frac{AB}{MN} = \frac{AC}{MP} = \frac{BC}{NP} \Rightarrow \frac{9}{MN} = \frac{AC}{22,5} = \frac{12}{18}$$

$$\frac{9}{MN} = \frac{12}{18} \Rightarrow MN = \frac{18 \cdot 9}{12} = \frac{27}{2} = 13,5$$

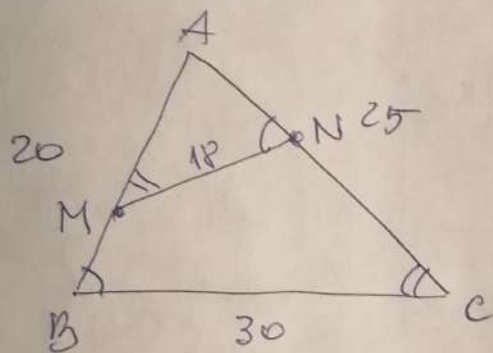
$$\frac{AC}{22,5} = \frac{12}{18} \Rightarrow AC = \frac{22,5 \cdot 12}{18} = 15$$

4) $\triangle ABC$
 $M \in (AB)$; $N \in (AC)$ a. p. $\widehat{ANM} \equiv \widehat{ABC}$

a) Demonstrați că $\triangle ANM \sim \triangle ABC$

b) Dacă $AB = 20$, $AC = 25$, $BC = 30$, $MN = 18$ calculați AM și AN .

Rezolvare



a) $\triangle ANM \equiv \triangle ABC$ } \Rightarrow ~~$\triangle ANM \sim \triangle ABC$~~
 $\angle A$ comun

$\Rightarrow \triangle AMN \sim \triangle ABC$

\Rightarrow o pereche de unghiuri
 congruente $\triangle AMN \equiv \triangle ACB$

b) $\Rightarrow \frac{AM}{AC} = \frac{AN}{AB} = \frac{MN}{BC} \Rightarrow \frac{AM}{25} = \frac{AN}{20} = \frac{18}{30}$

$\frac{AM}{25} = \frac{18}{30} \Rightarrow AM = \frac{25 \cdot 18}{30} = 15$

$\frac{AN}{20} = \frac{18}{30} \Rightarrow AN = \frac{20 \cdot 18}{30} = 12$

Jeme din sublegere (partea I)

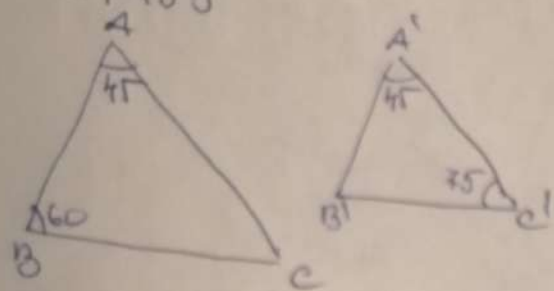
1 b, c / pag. 155

~~2 a / pag. 155~~

3 a, b / pag. 155

Тема - решалка

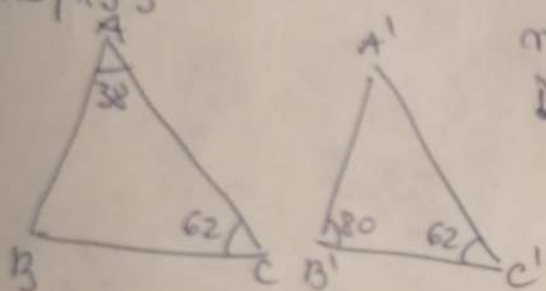
1b/155



$$m(\hat{B}') = 180 - (45 + 75) = 60$$

$$\text{Деци } \left. \begin{array}{l} \hat{B} \equiv \hat{B}' \\ \hat{A} \equiv \hat{A}' \end{array} \right\} \begin{array}{l} \text{UU} \\ \Rightarrow \Delta ABC \sim \Delta A'B'C' \end{array}$$

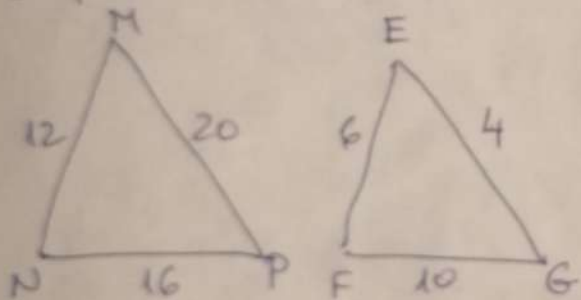
1c/155



$$m(\hat{B}') = 180 - (38 + 62) = 80$$

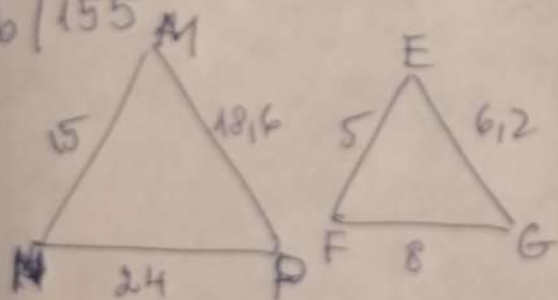
$$\text{Деци } \left. \begin{array}{l} \hat{B} \equiv \hat{B}' \\ \hat{C} \equiv \hat{C}' \end{array} \right\} \begin{array}{l} \text{UU} \\ \Rightarrow \Delta ABC \sim \Delta A'B'C' \end{array}$$

3a/155



$$\frac{12}{6} \neq \frac{16}{10} \neq \frac{20}{4} \Rightarrow \Delta ABC \not\sim \Delta EFG$$

3b/155



$$\left. \begin{array}{l} \frac{MN}{EF} = \frac{18,6}{5} = 3 \\ \frac{NP}{FG} = \frac{24}{8} = 3 \\ \frac{MP}{EG} = \frac{18,6}{6,2} = 3 \end{array} \right\} \begin{array}{l} \text{LLL} \\ \Rightarrow \end{array}$$

$$\Delta MNP \sim \Delta EFG$$